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# Determination of Hot-Gas Temperature Profiles from Infrared Emission and Absorption Spectra

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## I. Introduction

We have studied the relationships between the temperature distribution in a hot gas and the infrared emission and absorption spectra of the gas, with a view to determining gas temperature profiles from line-of-sight spectral measurements. Temperature gradients affect the exchange of radiant energy between different regions of a hot gas and, consequently, affect the observed spectra. As a result, it is possible to obtain information about the temperature distribution in a hot gas from a study of its emission and absorption spectra.

## II. Theory

### *Emittance equations*

When temperature gradients exist in a gas, it is possible to consider the gas to consist of a series of zones, each of which is isothermal within the precision of measurement. If the gradients are steep the regions would have to be small, but, in principle, such a division always can be made. If we number these zones serially from 1 to $n$, with zone 1 nearest the detector, the irradiance of the detector, $H(\lambda_j)$, by monochromatic radiation of wavelength $\lambda_j$, is given[1] by Eq. (1)

$$H(\lambda_j) = \sum_{i=1}^{n} W_b(\lambda_j, T_i)[1 - \tau_i(\lambda_j)] \prod_{h=0}^{i-1} \tau_h(\lambda_j) \qquad (1)$$

$W_b(\lambda_j, T_i)$ is the Planck function at wavelength $\lambda_j$ and the temperature $T_i$ of zone $i$. $\tau_i(\lambda_j)$ denotes the transmittance of zone $i$ at $\lambda_j$. The right side of Eq. (1) is simply the emittance of each zone $i$, as given by Kirchhoff's Law, multiplied by the transmittance of the gas between zone $i$ and the detector, summed over all of the zones. For monochromatic radiation, the transmittance of any number of zones is the product of the transmittances of the component zones. Zone zero is the medium between the sample and the detector.

In practice, the measured radiant energy is never truly monochromatic. Over the finite slit width $\Delta\lambda'$ of the monochromator used, the Planck function usually can be considered constant, but this is not true of the transmittance of a gas. If $g(\lambda_j, \lambda')$ symbolizes the spectrometer split function at $\lambda_j$, and measured quantities are labeled by a subscript $m$, we can write

$$\begin{aligned} H_m(\lambda_j) &= \int_{\Delta\lambda'} H(\lambda')g(\lambda_j, \lambda')d\lambda' \\ &= \sum_{i=1}^{n} W_{bm}(\lambda_j, T_i)[\bar{\tau}_{(i-1)}(\lambda_j) - \bar{\tau}_i(\lambda_j)] \qquad (2) \end{aligned}$$

where

$$\bar{\tau}_i(\lambda_j) = \frac{\int_{\Delta\lambda'} \prod_{h=0}^{i} \tau_h(\lambda')g(\lambda_j, \lambda')d\lambda'}{\int_{\Delta\lambda'} g(\lambda_j, \lambda')d\lambda'} \qquad (3)$$

Physically, $\bar{\tau}_i(\lambda_j)$ is the transmittance of the section of the sample, composed of zones 0–$i$, that would be measured at $\lambda_j$ with the slit function $g(\lambda_j, \lambda')$ and a spectral slit width $\Delta\lambda'$, and $W_{bm}(\lambda_j, T_i)$ is the spectral emittance of a blackbody at $T_i$ that would be measured at $\lambda_j$ with a slit function $g(\lambda_j, \lambda')$ and a spectral slit width $\Delta\lambda'$. Since the blackbody emittance is effectively constant over the values of $\Delta\lambda'$ corresponding to practical slit widths, $W_{bm}(\lambda_j, T_i)$ can be separated into two factors, one of which is independent of temperature, and the other is independent of slit function. Thus

$$W_{bm}(\lambda_j, T_i) = W_b(\lambda_j, T_i) \int_{\Delta\lambda'} g(\lambda_j, \lambda')d\lambda' \qquad (4)$$

Since $W_b(\lambda_j, T_i)$ is given by the Planck law, a measurement of the apparent emittance $W_{bm}(\lambda_j, T_b)$ of a blackbody at any convenient temperature $T_b$ can be used to evaluate

$$\int_{\Delta\lambda'} g(\lambda_j, \lambda')d\lambda'$$

Then, when Eq. (2) is solved for the values of $W_{bm}(\lambda_j, T_i)$, the temperatures can be obtained from the corresponding values of $W_b(\lambda_j, T_i)$, and the results will be independent of the slit function.

For an isothermal temperature profile, $n = 1$, and Eq. (2) therefore contains only one transmittance $\bar{\tau}(\lambda_j)$, provided that the optical path outside the hot-gas sample is free of absorbing gas, i.e., the path is evacuated or flushed with a nonabsorbing gas so that $\bar{\tau}_0(\lambda_j) = 1$. Moreover, $\bar{\tau}(\lambda_j)$ can be measured directly, as can $H_m(\lambda_j)$. Equation (2) then reduces to

$$H_m(\lambda_j) = W_{bm}(\lambda_j, T)[1 - \bar{\tau}(\lambda_j)] \qquad (2a)$$

Equation (2a) has the form of Kirchhoff's law. It can be solved for the only remaining variable $W_{bm}(\lambda_j, T)$ from which the temperature can be determined. This is the infrared brightness method of temperature measurement.[2] If Eq. (2a) is applied to a nonisothermal system, the wavelength-dependent result is a weighted average of the temperatures in the sample.

In applying Eq. (2) to a nonisothermal temperature profile, spectral emittance is measured at $n$ different wavelengths, having $n$ different sets of values of $\bar{\tau}_i(\lambda_j)$. This yields $n$ independent simultaneous equations for calculating the $n$ temperatures.

The infrared transmittances that appear in Eq. (2) are themselves somewhat temperature-dependent. Moreover, of the $n$ transmittances that must be obtained, only one $[\bar{\tau}_n(\lambda_j)]$ can be measured directly. Therefore, solution of Eq. (2) requires some a priori knowledge of the transmittances of the molecular species comprising the hot-gas specimen, particularly a knowledge of how they vary with temperature. This knowledge may take the form of empirical data, theoretical formulas, or a combination of these. Once such information is available, our system of irradiance equations can be solved for the thermal structure of the specimen. We have developed and tested the following iterative procedure for carrying out this solution.

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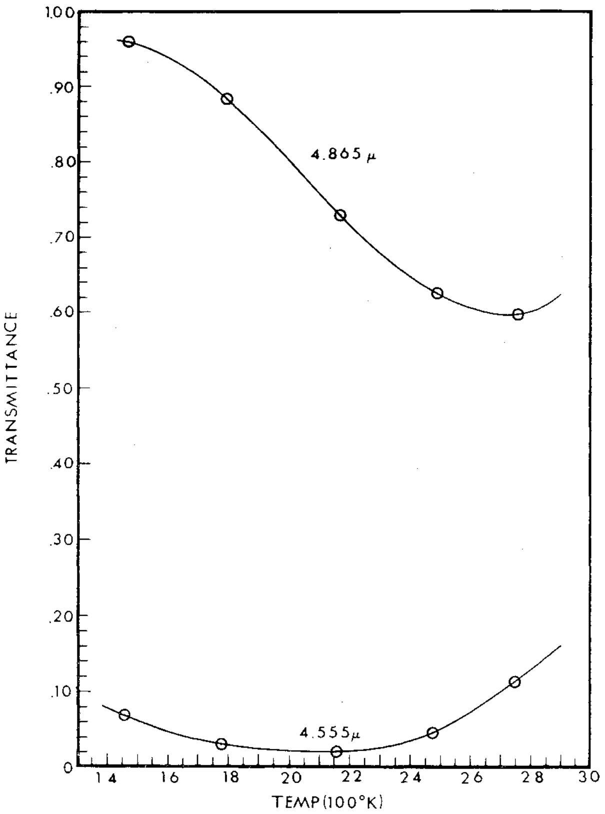

**Fig. 1 Zonal transmittance as a function of temperature.**

### *Method of solution*

$$\Delta W_b(\lambda_j, T_i) = W_b(\lambda^\circ, T_i) - W_b(\lambda_j, T_i) \tag{5}$$

$$H_m(\lambda_j) + \sum_{i=1}^{n} \Delta W_{bm}(\lambda_j, T_i)[\bar{\tau}_{(i-1)}(\lambda_j) - \bar{\tau}_i(\lambda_j)] = \sum_{i=1}^{n} W_{bm}(\lambda^\circ, T_i)[\bar{\tau}_{(i-1)}(\lambda_j) - \bar{\tau}_i(\lambda_j)] \tag{6}$$

Equation (5) defines a term $\Delta W_b(\lambda_j, T_i)$ as the difference between the Planck function at some convenient wavelength $\lambda^\circ$ in the general spectral region being studied and the Planck function at wavelength $\lambda_j$, at which an actual spectroscopic measurement is made. Solving this equation for $W_b(\lambda_j, T_i)$ and substituting the result into Eq. (2) yields Eq. (6). This is done using the same $\lambda^\circ$ for each of the $n$ equations to be solved. The thermal variations of the values of $W_{bm}(\lambda^\circ, T_i)$ are generally greater than those of either the transmittances or the values of $\Delta W_{bm}(\lambda_j, T_i)$. Therefore, one might hope for convergence of an iteration scheme in which transmittances and $\Delta W_{bm}(\lambda_j, T_i)$'s, determined from a rough estimate of the temperature profile, are used in the irradiance equations to calculate a set of $W_{bm}(\lambda^\circ, T_i)$'s from which a better estimate of the temperature profile may be obtained. More accurate transmittance and $\Delta W_{bm}(\lambda_j, T_i)$ values then can be computed and the procedure repeated until the calculated thermal structure stops changing appreciably with succeeding cycles.

**Table 1 Two-zone transmittance and emittance measurements**

Measured transmittances
$\bar{\tau}_2(4.555\text{-}\mu) = 0.01$
$\bar{\tau}_2(4.865\text{-}\mu) = 0.57$

Measured spectral emittance $= \dfrac{H_m(\lambda_j)}{\int_{\Delta\lambda'} g(\lambda_j, \lambda')d\lambda'} = W_m(\lambda_j)$

$W_m(4.555\text{-}\mu) = 2.83 \text{ w/cm}^2\mu$
$W_m(4.865\text{-}\mu) = 2.80 \text{ w/cm}^2\mu$

**Table 2 Temperature profile calculations**

| | $T_1$ | $T_2$ |
|---|---|---|
| Initial estimate | 1550°K | 2618°K |
| 1st cycle result | 1478°K | 2756°K |
| 2nd cycle result | 1466°K | 2745°K |
| 3rd cycle result | 1464°K | 2740°K |
| 4th cycle result | 1464°K | 2739°K |
| Infrared (IR) brightness temperatures of individual zones | 1460°K | 2740°K |

## III. Experiments and Results

The method has been tested by measurements of known temperature profiles of hot $CO_2$ produced by flames on flat flame burners. The measurements were made in the 4.3-μ region with an emission-absorption spectroradiometer system that provides an optical path completely flushed with dry nitrogen, with the exception of the sample space. In the burner assemblies, each zone consisted of a 2- × 2-in. burner, flanked by a pair of ½- × 2-in. burners. Hot $CO_2$ was generated by burning carbon monoxide with oxygen on the 2- × 2-in. burner and adding a controlled amount of cold $CO_2$ to each burning mixture to regulate the flame temperature. The flame propagation rate was adjusted by adding measured traces of hydrogen. "Guard" flames on the ½- × 2-in. burners produced combustion products that had the same temperature as the $CO_2$, but all of them were transparent at the wavelengths studied. Therefore, each zone was simply an isothermal specimen of $CO_2$ as far as the spectroscopic observations were concerned. Measurements were made on individual zones to obtain their transmittances and infrared brightness temperatures as well as on 2 zones in series to test methods of calculating temperature profiles from spectroscopic data.

The requisite a priori knowledge of zonal transmittance as a function of temperature was obtained from the graphs shown in Fig. 1. These curves were obtained by plotting measured transmittances of single zones at five temperatures between 1450° and 2800°K.

Measurements of spectral emittances and transmittances of a 2-zone flame assembly in which the individual zones 1 and 2 had infrared brightness temperatures of 1460° and 2740°K, respectively, are shown in Table 1. The temperature profile of the assembly was calculated, assuming that we had no specific information about the system except for the composition, pressure, and the four measurements given in Table 1.

From the data in Table 1, two infrared brightness temperatures were calculated, each giving a weighted average of the temperatures in the various regions of the specimen. The one at 4.555 μ, where the transmittance was very low, would weight zone 1 very heavily. Therefore, this infrared brightness temperature was used for an initial estimate of $T_1$. The infrared brightness temperature calculated at 4.865 μ would give more weight to zone 2, so it was used for an initial approximation to $T_2$.

The temperature profile then was calculated by means of the iteration scheme previously described. Convergence was achieved in 4 cycles, as shown by Table 2. The calculated results agreed very well with the zonal temperatures determined by the infrared brightness method.

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